Sub-Exposure Times and Dark Frames

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Note on Revision 2

Dr. Jeff Winter was kind enough to provide additional insight on the nature of combining correlated and uncorrelated noise sources, which resulted in a substantial change to this paper. When dark frames are combined and the resultant "master" dark is subtracted from the individual data frames in a stack, the master dark is correlated. The same dark, albeit lower noise, is subtracted from each data frame. Thus, just as taking more sub-exposures increases the SNR and makes faint data visible, so too the noise from the master dark is reinforced and becomes more visible. As a result, the size of the stack is an explicit factor in determining the number of dark frames required for a specified noise contribution from dark subtraction.

Introduction

As a follow-up to my earlier paper on Sub-Exposure Time paper, I have incorporated some analytical work done by Stan Moore in factoring the impact of dark frames on the noise in the SNR Equation. Many imagers take way more dark frames than necessary. While no real damage is done, some time is wasted. The number of dark frames you should take depends on a number of parameters and this paper will discuss them.

Discussions

Repeating the original SNR Equation:

$$(1) \qquad SNR = \sqrt{N}\,\frac{E_{obj}t}{\sqrt{E_{obj}t + E_{sky}t + N_d + R_{on}^2}}$$

Where

N = number of sub-exposures and assumes a mean combine.
E_{obj} = Object flux in electrons per second
E_{sky} = Sky background flux in electrons per second
t = Exposure time in seconds
N_d = Number of dark current electrons
R_{on} = Readout noise in electrons

The dark signal is a function of the individual chip and is specified by two parameters"

D_e, the dark signal, usually specified e/pixel/sec. and X_e, the doubling rate of the dark signal, expressed in °C. This is the temperature difference that causes the dark signal to increase with increasing temperature or decrease with decreasing temperature. De is usually specified at a specific temperature. So, we can express the dark signal flux (electrons/sec.) as

(2) $$N_e = \frac{D_e}{2^{\frac{T_s - Tc}{Xe}}} \text{ and therefore } N_d = N_e t$$

Where

D$_e$ = the nominal dark signal at T$_s$
T$_s$ = the temperature at which D$_e$ is specified
T$_c$ = the CCD operating temperature when cooled
X$_e$ = the dark signal doubling rate

D$_e$, T$_s$ and X$_e$ are unique to and specified for a given CCD sensor.

D$_e$ is normally measured by the sensor vendor by averaging the pixel levels over the entire sensor area. In many cases, this may actually *overstate* the actual dark current level, since hot pixels may skew the average higher.

Assuming a direct subtract of a master dark frame resulting from a mean combine, equation (1) becomes:

(3) $$SNR = \sqrt{N}\frac{E_{obj}t}{\sqrt{E_{obj}t + E_{sky}t + N_e t + R_{on}^2 + \frac{N(N_e t + R_{on}^2)}{m}}}$$

Where

m is the number of dark frames mean combined.

We can determine a suitable sub-exposure time using the results of the previous paper. We can then assess the implications of multiple dark frames by determining the number of dark frames required to meet a maximum contribution of the master dark frame to the overall noise.

Let q be the maximum allow increase in noise, due to the master dark frame. We can then write

(4) $$(1+q)^2(E_{sky}t + N_e t + R_{on}^2) = E_{sky}t + N_e t + R_{on}^2 + \frac{N(R_{on}^2 + N_e t)}{m}$$

We can then solve this equation for m

(5) $$m = \frac{N(R_{on}^2 + N_e t)}{[(1+q)^2 - 1](E_{sky}t + R_{on}^2 + N_e t)}$$

An Example

We can compare two popular image sensors to see the impact of their noise performance on the number of dark frames required. The KAI11002 is in wide use and the KAF16803 represents current performance of low dark signal sensors. Here are the specifics:

Parameter	KAI11002	KAF16803
Dark signal @ Test Temperature, T_s	880 @ 40°C	15 @ 25°C
Dark signal doubling temperature, X_e	6.3°C	6.3°C
Read noise, R_{on}	12e	11e

Table 1: Sensor Parameters

Let's assume a sky flux of 3 e/sec., a reasonable number for a suburban sky. This is my local sky flux. While it might be expected that the KAI11002, having a lower overall QE might be smaller, we will use the same sky flux for this example. Assume an exposure time of 600 sec. and a stack of 10 sub-exposures. Let's allow the dark frame subtraction to contribute a 5% increase to the overall noise. Calculating m for both sensors at different cooler temperatures gives the following data:

	-20°C	-30°C	-40°C	-50°C
KAI11002	32	18	11	9
KAF16803	10	8	7	7

Table 2: Number of Mean-combined Dark Frames

Is this reasonable? Let's look at the noise in the dark signal over the same temperature range and compare it to typical read noise.

	Read noise	-20°C	-30°C	-40°C	-50°C
KAI11002	12	26.8	15.4	8.9	5.1
KAF16803	11	8.0	4.6	2.7	1.5

Table 3: Noise from Dark Signal

In the case of the 11002, we need enough dark exposures to drive down the noise form the dark signal *and* the read noise; with the 16803, the dark signal rapidly decreases so the predominant noise source is read noise. In fact, for the 16803 operating at -30°C or colder, it could be argued that subtracting a master bias frame is more than sufficient.

Application

We need to evaluate the terms in equation (5) to get a value for m. For N_e, we need the terms of equation (2) above. The value for p is determined by our choice in this equation:

$$(6) \qquad t_{ORN} = \frac{R_{on}^2}{[(1+p)^2 - 1]E_{sky}}$$

Where

t_{orn} is the calculated overwhelm read noise exposure time, based on how much we want to overwhelm it.

E_{sky} can be determined by taking a few minute exposure of duration $t_{test.}$ If you take an autodark immediately after the light exposure, you will remove any bias level. Measure the average background of the resulting image and call this $ADU_{background}$. E_{sky} is then calculated based on this data as follows:

$$(7) \qquad E_{sky} = \frac{(ADU_{background} - 100)g}{t_{test}}$$

100 should be subtracted from the background ADU if your camera control program adds a 100 count pedestal to prevent negative numbers. Many programs do.

So we now have everything we need to calculate the number of mean-combined dark frames according to what noise percentages we want both the readout noise and the master dark frames to contribute. A sub-exposure of at least t_{orn} seconds should be used. But there is one more factor we should consider – cosmic ray impacts.

We know that cosmic rays "happen" and can add spurious bright spots to the dark frame. If we mean combine the individual dark frames, these cosmic rays, which are typically quite energetic and therefore have high ADU values, will be reduced only be the averaging mechanism. In other words, if there are m dark frames, and the cosmic ray impact is in a unique position on the chip for only one frame, the resultant ADU of the cosmic ray becomes ADU/m.

Other combination techniques are used to more effectively remove cosmic ray effects. One is a median combine and another is a min-max clip mean combine. Both of these techniques are discussed in the companion paper, together with their noise combination effects,

Thus our calculated mean needs to be adjusted for the planned combine method. For example, a median combine will require

$\frac{\pi}{2}m + 1$ dark frames

and min-max clip mean combine will require $m + 2$ dark frames. Min-max clip is a much more efficient method than median combine for removing these defects. See the spreadsheet referenced below for comparative counts.

A spreadsheet has been developed to facilitate the application of these concepts. Exposure times are calculated for t_{orn} as defined in this companion paper:
http://www.hiddenloft.com/notes/SubExposures.pdf
The spreadsheet is available at:
http://www.hiddenloft.com/notes/DarkandSubExp2.xls
These calculations are even easier to do using the Tools page of CCDAutoPilot version 4. See
http://www.ccdware.com/products/ccdap4/

Instructions for use are provided in the spreadsheet.

Conclusion

I hope this gives you a strategy for both your sub-exposures and the number of dark frames you should use to achieve a desired noise contribution from both read noise and the master dark frame.

One concluding note: This analysis is based on direct subtract of a master dark whose exposure time is equal to that of the data frame. A similar analysis scaled darks would be required. Scaling a dark can only add more noise, since the bias must be subtracted from both the data frame and the master dark frame before scaling. All of these calculations will add noise, generally, but not always in the form of readout noise. If the lowest noise data is desired, scaled darks should be avoided, unless a *very* large number of dark and bias frames are used to create the masters.