

# Sub-Exposure Times and Dark Frames

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## Introduction

As a follow-up to my earlier paper on Sub-Exposure Time paper, I have incorporated some recent analytical work done by Stan Moore in factoring the impact of dark frames on the noise in the SNR Equation. Many imagers take way more dark frames than necessary. While no real damage is done, some time is wasted. The number of dark frames you should take depend on a number of parameters and this paper will discuss them.

## Discussions

Repeating the original SNR Equation:

$$(1) \quad SNR = \sqrt{N} \frac{E_{obj}t}{\sqrt{E_{obj}t + E_{sky}t + N_d + R_{on}^2}}$$

Where

N = number of sub-exposures and assumes a mean combine.

$E_{obj}$  = Object flux in electrons per minute

$E_{sky}$  = Sky background flux in electrons per minute

t = Exposure time in minutes

$N_d$  = Number of dark current electrons

$R_{on}$  = Readout noise in electrons

The dark signal is a function of the individual chip and is specified by two parameters”

$D_e$ , the dark signal, usually specified e/pixel/sec. and  $X_e$ , the doubling rate of the dark signal, expressed in °C. This is the temperature difference that causes the dark signal to increase with increasing temperature or decrease with decreasing temperature.  $D_e$  is usually specified at a specific temperature. So, we can express the dark signal flux as

$$(2) \quad N_e = \frac{D_e}{2^{\frac{T_s - T_c}{X_e}}}$$

Where

$D_e$  = the nominal dark signal at  $T_s$

$T_s$  = the temperature at which  $D_e$  is specified

$T_c$  = the CCD operating temperature when cooled

$X_e$  = the dark signal doubling rate

$D_e$ ,  $T_s$  and  $X_e$  are unique to and specified for a given CCD sensor.

$D_e$  is normally measured by the sensor vendor by averaging the pixel levels over the entire sensor area. In many cases, this may actually *overstate* the actual dark current level, since hot pixels may skew the average higher.

Assuming a direct subtract of a master dark frame resulting from a mean combine, equation (1) becomes:

$$(3) \quad SNR = \sqrt{N} \frac{E_{obj}t}{\sqrt{E_{obj}t + E_{sky}t + N_e t + R_{on}^2 + \frac{N_e t + R_{on}^2}{m}}}$$

Where

$m$  is the number of dark frames mean combined.

We can determine a suitable sub-exposure time using the results of the previous paper. We can then assess the implications of multiple dark frames by determining the number of dark frames required to meet a maximum contribution of the master dark frame to the overall noise.

Let  $q$  be the maximum allow increase in noise, due to the master dark frame. We can then write

$$(4) \quad (1 + q)^2 (E_{sky}t + N_e t + R_{on}^2) = E_{sky}t + N_e t + R_{on}^2 + \frac{R_{on}^2 + N_e t}{m}$$

We can then solve this equation for  $m$

$$(5) \quad m = \frac{R_{on}^2 + N_e t}{[(1 + q)^2 - 1](E_{sky}t + R_{on}^2 + N_e t)}$$

## Application

We need to evaluate the terms in equation (5) to get a value for  $m$ . For  $N_e$ , we need the terms of equation (2) above. The value for  $p$  is determined by our choice in this equation:

$$(6) \quad t_{ORN} = \frac{R_{on}^2}{[(1 + p)^2 - 1]E_{sky}}$$

Where

$T_{orn}$  is the calculated overwhelm read noise exposure time, based on how much we want to overwhelm it.

$E_{sky}$  can be determined by taking a few minute exposure of duration  $t_{test}$  and then measuring the average background of the resulting image and calling this  $ADU_{background}$ .  $E_{sky}$  is then calculated based on this data as follows:

$$(7) \quad E_{sky} = \frac{(ADU_{background} - 100)g}{t_{test}}$$

100 should be subtracted from the background ADU if your camera control program adds a 100 count pedestal to prevent negative numbers. Many programs do.

So we now have everything we need to calculate the number of mean-combined dark frames according to what noise percentages we want both the readout noise and the master dark frames to contribute. But there is one more factor we should consider – cosmic ray impacts.

We know that cosmic rays “happen” and can add spurious bright spots to the dark frame. If we mean combine the individual dark frames, these cosmic rays, which are typically quite energetic and therefore have high ADU values, will be reduced only by the averaging mechanism. In other words, if there are  $m$  dark frames, and the cosmic ray impact is in a unique position on the chip for only one frame, the resultant ADU of the cosmic ray becomes  $ADU/m$ .

Other combination techniques are used to more effectively remove cosmic ray effects. One is a median combine and another is a min-max clip mean combine. Both of these techniques are discussed in the companion paper, together with their noise combination effects,

Thus our calculated mean needs to be adjusted for the planned combine method. For example, a median combine will require

$$\frac{\pi}{2} m \text{ dark frames}$$

and min-max clip mean combine will require  $m + 2$  frames.

A spreadsheet has been developed to facilitate the application of these concepts. Exposure times are calculated for both the  $t_{om}$  and the  $t_{om}/2$  case, as defined in this companion paper:

<http://www.hiddenloft.com/notes/SubExposures.pdf>

The spreadsheet is available at:

<http://www.hiddenloft.com/notes/DarkandSubExp.xls>

Instructions for use are provided in the spreadsheet.

## Conclusion

I hope this gives you a strategy for both your sub-exposures and the number of dark frames you should use to achieve a desired noise contribution from both read noise and the master dark frame.

One concluding note: This analysis is based on direct subtract of a master dark whose exposure time is equal to that of the data frame. A similar analysis for this situation would be required. Scaling a dark can only add more noise, since the bias must be subtracted from both the data frame and the master dark frame before scaling. All of these calculations will add noise, mostly

in the form of readout noise. If the lowest noise data is desired, scaled darks should be avoided, unless a very large number of dark and bias frames are used to create the masters.